Chapter 4

ELEMENTARY DEFINITIONS and THE KINEMATIC EQUATIONS

A.) Speed:

1.) Average speed (s_{avg}): A scalar quantity that denotes the *average distance traversed per unit time* (i.e., the average rate at which ground is covered). With units of *meters per second*, it is mathematically defined as:

$$s_{avg} = \frac{\Delta d}{\Delta t},$$

where Δd is the total-distance-traveled during a time interval Δt.

a.) Example: A running woman covers 100 meters in 15 seconds, then changes direction and hops 30 meters in 10 seconds (see Figure 4.1). What is her *average speed* for the overall motion?

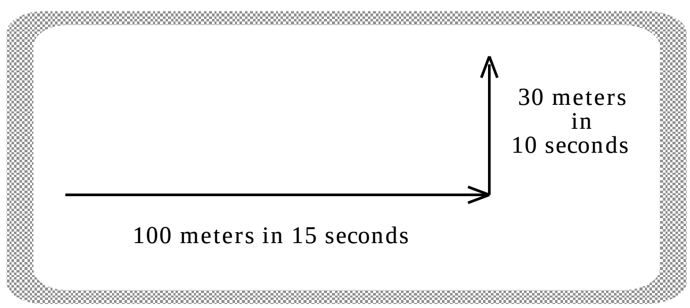

FIGURE 4.1

$$s_{avg} = \Delta d / \Delta t$$
$$= (130 \text{ m}) / (25 \text{ sec})$$
$$= 5.2 \text{ m/s}.$$

Note 1: What this average gives you is the SINGLE CONSTANT SPEED that will move the woman over the required distance (130 meters) in the required time (25 seconds). It does not tell you how fast she is actually traveling at any given instant. She could run the first 80 meters in 10 seconds, then stand panting for 2 seconds, then do the last 20 meters of the first leg in the remaining 3 seconds. *Average speed* tells you nothing about the actual motion; all it tells you is the single speed that would be required to go the distance at a uniform run in the allocated time.

Note 2: *Speed* is not a quantity physicists use very much. It is being presented here as a preamble to more interesting and useful variables to come.

2.) Instantaneous speed (s): A measure of an object's *distance traveled per unit time* (i.e., its rate of travel), measured at a particular point in time.

a.) Example: The running woman in Example 1a is found to be moving with a speed of 8 m/s as she passes the 15 meter mark, three tenths of a second into the race. Her *instantaneous speed* at the 15 meter mark is, therefore, 8 m/s.

b.) Mathematically, *instantaneous speed* (referred to simply as *speed* from here on) is defined as:

$$s = \text{limit}_{\Delta t \Rightarrow 0} \left(\Delta d / \Delta t \right).$$

Note: Translation: At a particular point in time, an object's *instantaneous speed* is equal to its *average speed* calculated over a very tiny time interval (i.e., as Δt approaches zero). Although this is technically a Calculus problem (we are actually looking at the *time derivative* of the distance function), it will not be written in that form. Again, the idea of *speed* is useful as a concept only. We will rarely use it as a mathematical entity.

B.) Velocity--Magnitude and Direction:

1.) Average velocity ($\mathbf{v}_{avg}$): A *vector* quantity that denotes the *average displacement* (i.e., the *net resultant* change of position) *per unit time* over some large time interval. With units of *meters per second* (normally written as m/s), it is mathematically defined as:

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t},$$

where $\Delta \mathbf{r}$ is the NET DISPLACEMENT of the object during a time interval Δt. The direction of $\mathbf{v}_{avg}$ is the same as the direction of $\Delta \mathbf{r}$, (i.e., that of the *net displacement*).

a.) Example: A woman covers 100 meters in 15 seconds, then changes direction and hops 30 meters in 10 seconds (see Figure 4.2). What is her *average velocity* for the overall motion?

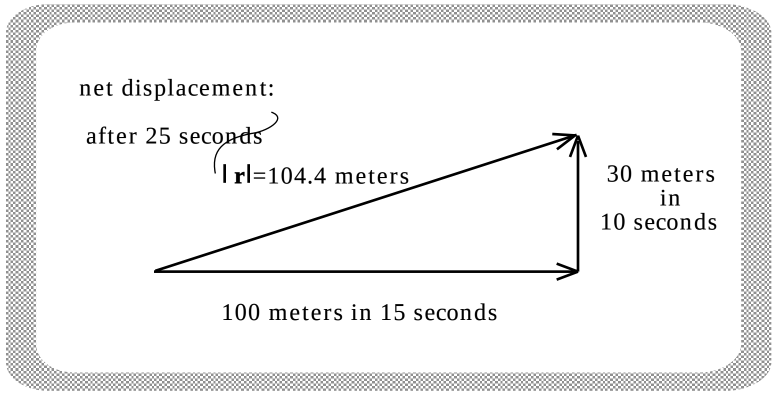

FIGURE 4.2

The magnitude:

$$\left|\mathbf{v}_{avg}\right| = \left|\frac{\Delta \mathbf{r}}{\Delta t}\right|$$
$$= (104.4 \text{ m})/(25 \text{ sec})$$
$$= 4.176 \text{ m/s}.$$

The direction (using trig. and the sketch):

$$\phi = \tan^{-1} [(30 \text{ m})/(100 \text{ m})]$$
$$= 16.7^o.$$

As a vector:

$$\mathbf{v}_{avg} = 4.176 \text{ m/s} \angle 16.7^o.$$

Note 1: This average value gives you the constant number of meters-per-second, moving DIRECTLY from the initial to the final position, required to effect the net displacement in the allotted time. As was the case with average speed, it does not reflect the *actual* velocity of the woman at any particular instant.

Note 2: *Average velocity* is not a quantity physicists use very much, but *instantaneous velocity* is!

2.) Instantaneous velocity (**v**): A measure of an object's *displacement per unit time* as measured at a particular point in time.

a.) Mathematically, *instantaneous velocity* (referred to as *velocity* from here on) is defined as:

$$\mathbf{v} = \text{limit}_{\Delta t \Rightarrow 0} \left(\Delta \mathbf{r} / \Delta t \right)$$

where the direction of $\Delta \mathbf{r}$ is the direction of motion at a given instant. As this is the definition of a derivative, we can write the relationship as:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}.$$

b.) Example: A body's position function is $\mathbf{r}(t) = [(7k_1 t^3)\mathbf{i} - (4k_2/t)\mathbf{j}]$ *meters* (the *k's* are added for the sake of units--we will ignore them from here on--i.e., set their magnitudes at *one*). What is $\mathbf{v}(t)$?

Solution:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$
$$= \frac{d\left[(7t^3)\mathbf{i} - (4/t)\mathbf{j}\right]}{dt} \ \frac{\text{meters}}{\text{sec}}$$
$$= \left[(21t^2)\mathbf{i} + (4/t^2)\mathbf{j}\right] \text{ m/s.}$$

Note: THIS IS IMPORTANT. The *sign* of the velocity tells you the direction of motion of the body at a particular point in time. That is, if the velocity is *-3**j** m/s*, the body is moving in the *-y direction*.

3.) *Velocity* and the *POSITION vs. TIME* Graph:

a.) Consider the *POSITION vs. TIME* graph shown in Figure 4.3 to the right. *The slope of the tangent to the curve at a time t_1* gives us *the change of position with time at that point* (i.e., the velocity at that point). By definition, that slope equals the derivative of the position function (*dx/dt*) evaluated at t_1.

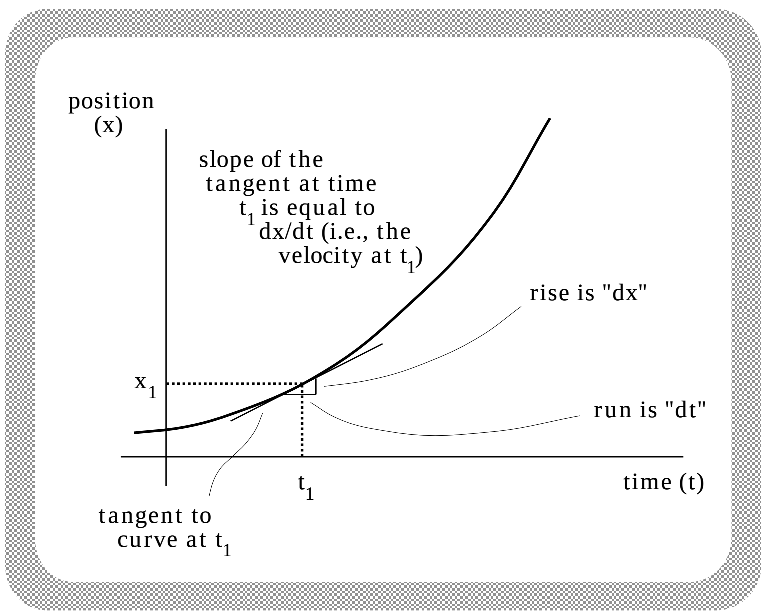

FIGURE 4.3

b.) Bottom line on *POSITION vs. TIME* graphs: To get the instantaneous velocity of a body whose POSITION vs. TIME graph is given but whose position *x(t)* is not explicitly known, draw a tangent to the curve at the time of interest, then determine the slope of that tangent. The slope will numerically equal the velocity of the body at that point in time.

4.) *Displacement* and the *VELOCITY vs. TIME* Graph:

a.) As was discussed in Chapter 3, the area under a *Velocity vs. Time* graph equals the *net displacement* of a body over the time interval in question (see Figure 4.4).

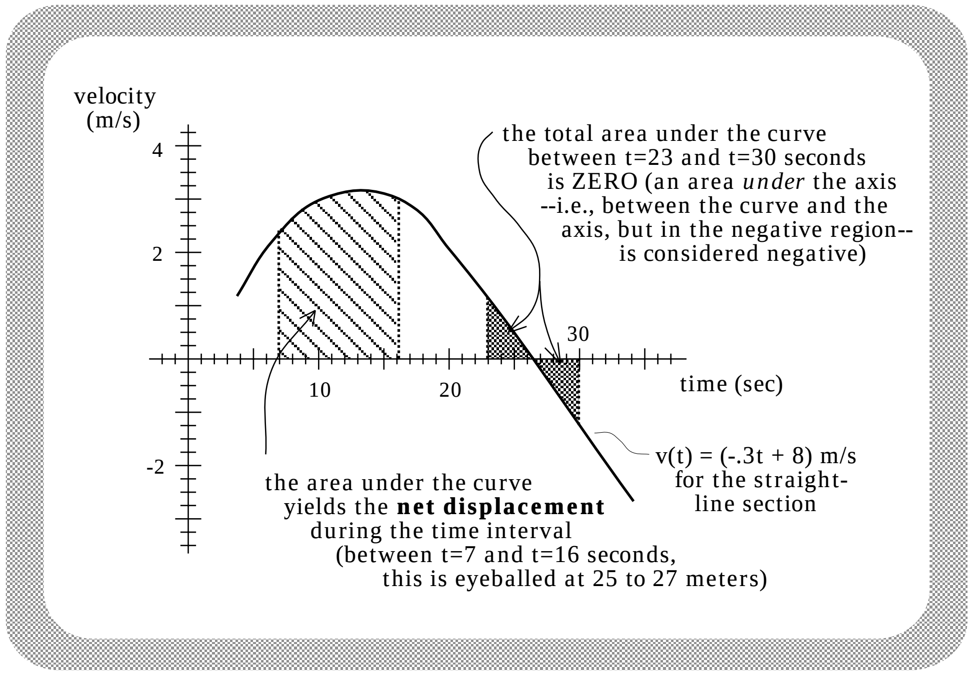

FIGURE 4.4

b.) In general, if we are given a velocity curve without an explicit function for the velocity (i.e., *v(t)*), we can find the *distance traveled* (i.e., Δx) by eyeballing the area under the curve over the time interval.

Note: Velocities under the axis (i.e., in the negative region) denote motion in the negative direction. That means a displacement associated with an area found *under the axis* is associated with *negative* displacement.

c.) If one knows the velocity function *v(t)*, the area under the curve (i.e., Δx) can be determined by integrating the velocity function over the time interval (i.e., by executing $\int(v)dt$).

d.) Example from the graph: Approximating the velocity function that defines the *straight-line section* of the graph as presented below:

$$v(t) = (-.3t + 8) \text{ m/s},$$

what is the area under the curve between times *t = 23 seconds* and *t = 30 seconds*? That is the same as asking, "What is the *net displacement* of the body during the time interval?"

Note: If our graph is to be believed, the answer to this question had better be zero! How so? The displacement between *t = 23 seconds* and *t = 26.5 seconds* is positive (that is, the *velocity* is positive so the body's *displacement* is in the *positive x direction*). On the other hand, the displacement between *t = 26.5 seconds* and *t = 30 seconds* is negative. Due to the symmetry of the situation, the two areas had better add up to zero.

i.) Integrating to determine the area under the curve between the times t_1 and t_2, we get:

$$
\begin{aligned}
\Delta x &= \int_{t=23}^{30} (-.3t + 8)dt \\
&= \int_{t=23}^{30} (-.3t)dt + \int_{t=23}^{30} (8)dt \\
&= \left[-.3\left(\frac{t^2}{2}\right) + 8t \right]_{t=23 \text{ sec}}^{30} \\
&= \left[-.3\left(\frac{(30)^2}{2}\right) + 8(30) \right] - \left[-.3\left(\frac{(23)^2}{2}\right) + 8(23) \right] \\
&= 105 - 104.65 \\
&= 0.35 \\
&\approx 0.
\end{aligned}
$$

C.) Acceleration--Magnitude and Direction:

1.) Average acceleration ($\mathbf{a}_{avg}$): A *vector* quantity that denotes the average *change-of-velocity per unit time* over some large time interval. Its units are *meters per second per second* (usually written as m/s^2--see Note #2 below), and its mathematical definition is:

$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t},$$

where $\Delta \mathbf{v}$ is the net *change of velocity* during a time interval Δt.

The *direction* of $\mathbf{a}_{avg}$ is the same as that of $\Delta \mathbf{v}$.

Note 1: Although it may not be obvious now, the *sign* of an acceleration value tells us information that is not immediately obvious. Explanation later!

Note 2: (concerning acceleration's units): The fraction (1/3)/3 can be re-written as $\frac{(1/3)}{(3/1)}$. Bringing the denominator up into the numerator by flipping it over and multiplying, we get (1/3)(1/3), or $1/3^2$. By the same token, as acceleration measures the rate at which velocity changes per unit time, its units are the ratio (m/s)/s. Being analogous to (1/3)/3, this can be written as m/s^2.

a.) Example: A man has velocity $\boldsymbol{v_1} = (3\ m/s)\boldsymbol{i}$ when at x_1. Three seconds later, he is at x_2 moving with velocity $\boldsymbol{v_2} = (9\ m/s)\boldsymbol{i}$ (see Figure 4.5). What is his *average acceleration*?

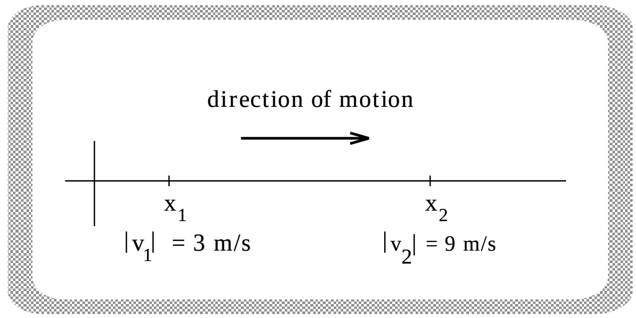

FIGURE 4.5

$$\begin{aligned}\mathbf{a}_{avg} &= \Delta\mathbf{v} / \Delta t \\ &= (\mathbf{v}_2 - \mathbf{v}_1) / (\Delta t) \\ &= (9\text{ m/s} - 3\text{ m/s})\mathbf{i} / (3\text{ sec}) \\ &= (2\ \mathbf{i})\text{ m/s}^2.\end{aligned}$$

2.) <u>Instantaneous acceleration</u> (**a**): A measure of an object's *change-of-velocity per unit time* <u>at a particular point in time</u>.

a.) Mathematically, instantaneous acceleration (referred to as *acceleration* from here on) is defined as:

$$\mathbf{a} = \text{limit}_{\Delta t \Rightarrow 0} \left(\Delta\mathbf{v} / \Delta t\right),$$

where the direction of $\Delta \boldsymbol{v}$ is the direction of the net force acting on the body at a given instant. As this is the definition of a derivative, we can write the relationship as:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}.$$

b.) Example: If a body's velocity is $\boldsymbol{v}(t) = [(21t^2)\boldsymbol{i} + (4/t^2)\boldsymbol{j}]\ m/s$, (ignoring units-constants) what is the body's acceleration $\boldsymbol{a}(t)$?

Solution:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$
$$= \frac{d\left[(21t^2)\mathbf{i} + (4/t^2)\mathbf{j}\right]}{dt}\left(\frac{m/s}{s}\right)$$
$$= \left[(42t)\mathbf{i} - (8/t^3)\mathbf{j}\right]\ m/s^2.$$

3.) *Acceleration* and the *VELOCITY vs. TIME* Graph:

a.) Consider the *VELOCITY vs. TIME* graph shown in Figure 4.6 to the right. The *slope of the tangent to the curve* at time t_1 is the *change of velocity with time* at that point (i.e., the acceleration at that point). By definition, that slope equals the derivative of the velocity function (*dv/dt*), evaluated at t_1.

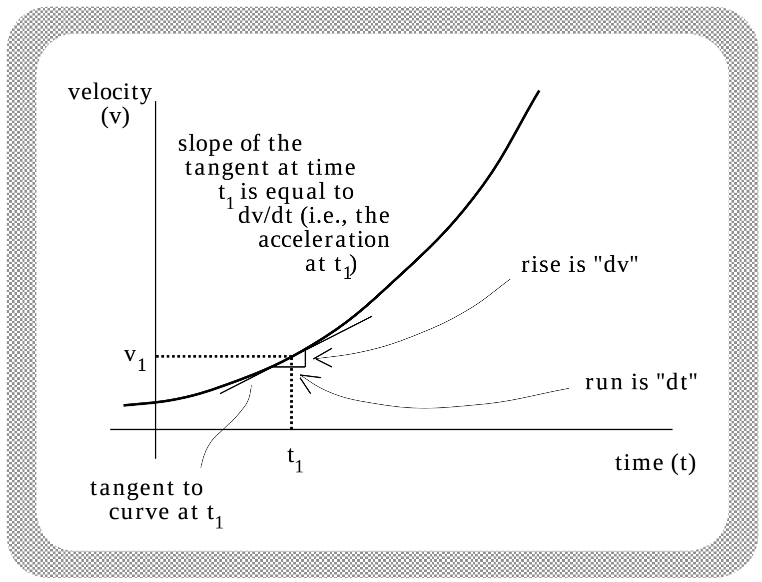

FIGURE 4.6

b.) Bottom line on *VELOCITY vs. TIME* graphs: To get the *instantaneous acceleration* of a body whose *VELOCITY vs. TIME* graph is given but whose velocity function *v(t)* is not explicitly known, draw a tangent to the curve at the time of interest, then determine the *slope of that tangent*. The slope will be numerically equal to the acceleration of the body at that point in time.

Note: If we have an *ACCELERATION vs. TIME* graph, the area under the graph between times t_1 and t_2 equals the *velocity change* during that time interval, and the *slope of the tangent* to the graph defines the *rate of change of acceleration with time*. This latter quantity is called *the jerk* of the motion (tough to believe, but true).

D.) Sign Significance for VELOCITY and ACCELERATION:

1.) Sign of the *VELOCITY* vector:

a.) It has already been noted that the direction of the *velocity* vector is the same as the direction of motion. A quick example follows:

b.) Consider an object moving along the *x axis*. It is initially found at $x_3 = 3$ *meters*; three seconds later it is found at $x_4 = -7$ *meters* (see Figure 4.7). What is the *average velocity* of the motion?

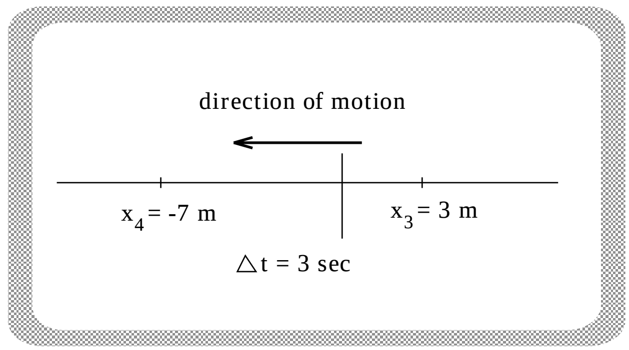

FIGURE 4.7

Solution:

$$\begin{aligned} \mathbf{v}_{avg} &= \Delta \mathbf{r} / \Delta t \\ &= \Delta \mathbf{x} / \Delta t \\ &= (x_4 - x_3)\mathbf{i} / \Delta t \\ &= [(-7 \text{ m}) - (+3 \text{ m})]\mathbf{i} / (3 \text{ sec}) \\ &= (-3.33\ \mathbf{i}) \text{ m/s}. \end{aligned}$$

Note: It is important to include *negative signs* when dealing with position variables.

c.) When the body is moving in the *-x direction*, the direction of the velocity vector is, indeed, in the *-**i** direction*.

Note: Even though -30 is smaller than +2 on a number line, the *sign of a velocity* has nothing to do with *magnitude* (i.e., how fast you are going)--all it tells you is *which way* you are going. (To see this: which would you prefer-- to be hit by a car moving with a velocity of +2 m/s or -30 m/s?)

2.) Sign of the *ACCELERATION* vector:

Note: Warning! You are about to find that the information wrapped up in the *sign* of an *acceleration* quantity is considerably more complicated than the information wrapped up in the sign of a *velocity* quantity.

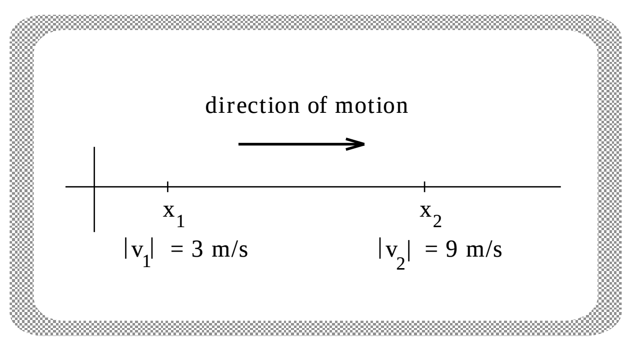

FIGURE 4.8

a.) A woman finds she is moving in the *+x direction* with velocity $v_1 = 3$

m/s. Three seconds later, she is moving with velocity $v_2 = 9\ m/s$ (see Figure 4.8 on the previous page). What is her *average acceleration*?

Note: Because we are working in one dimension only, we will not bother carrying the unit vector ***i*** along in the calculation.

$$\begin{aligned} \mathbf{a}_{avg} &= \Delta \mathbf{v} / \Delta t \\ &= (v_{sec\ pt} - v_{first\ pt}) / (\Delta t) \\ &= (v_2 - v_1) / (\Delta t) \\ &= (9\ m/s - 3\ m/s)/(3\ sec) \\ &= +2\ m/s^2. \end{aligned}$$

Observation 1: An individual *speeding up* while moving in the *+x direction* has a *POSITIVE acceleration*.

b.) A woman finds she is moving in the *+x direction* with velocity $v_3 = 9\ m/s$. Three seconds later, she is moving with velocity $v_4 = 3\ m/s$ (see Figure 4.9). What is her average acceleration?

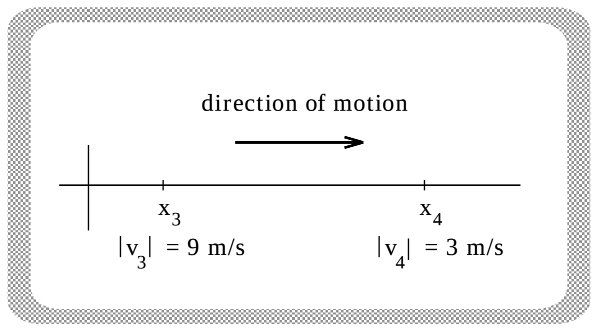

FIGURE 4.9

$$\begin{aligned} \mathbf{a}_{avg} &= \Delta \mathbf{v} / \Delta t \\ &= (v_4 - v_3) / (\Delta t) \\ &= (3\ m/s - 9\ m/s)/(3\ sec) \\ &= -2\ m/s^2. \end{aligned}$$

Observation 2: An individual *slowing down* while moving in the *+x direction* has a *NEGATIVE acceleration*.

Note: The combination of Observations 1 and 2 normally leads people to believe that *speeding up* is associated with positive acceleration (often referred to simply as acceleration) and *slowing down* is associated with negative acceleration (often called deceleration). THIS IS NOT ALWAYS THE CASE, as will be shown below.

c.) A woman moves in the *-x direction* with velocity $v_5 = 3\ m/s$. Three seconds later, she is found to be moving at velocity $v_6 = 9\ m/s$. What is her average acceleration?

Note: THERE IS SOMETHING RADICALLY WRONG WITH THE STATEMENT OF THIS PROBLEM. Can you find the error?

The problem should be stated: A woman finds she is moving in the *-x direction* with velocity $v_5 = -3\ m/s$. Three seconds later, she is found to be moving at velocity $v_6 = -9\ m/s$ (see Figure 4.10). What is her *average acceleration*?

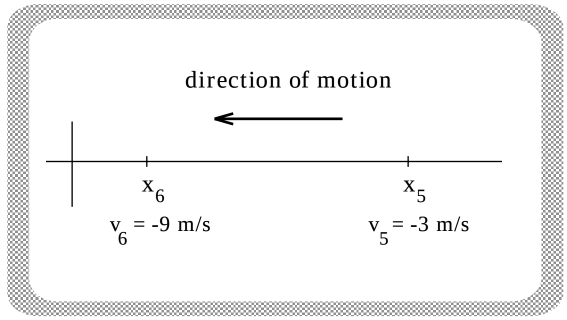

FIGURE 4.10

Note: The "RADICAL PROBLEM" alluded to above has to do with signs. BE CAREFUL WITH YOUR SIGNS; YOU WILL RARELY IF EVER WORK WITH TRUE, SIGNLESS MAGNITUDES. The sign of the *velocity* of an object moving in the *-x direction* is *negative*!

Solving the problem:

$$\begin{aligned} \mathbf{a}_{avg} &= \Delta \mathbf{v} / \Delta t \\ &= (v_6 - v_5) / (\Delta t) \\ &= [(-9\ m/s) - (-3\ m/s)]/(3\ sec) \\ &= -2\ m/s^2. \end{aligned}$$

Observation 3: Here we have a NEGATIVE ACCELERATION, but the woman isn't slowing down--she's *speeding up*.

Likewise, if the woman is moving in the *-x direction* with velocity $v_7 = -9\ m/s$ and, three seconds later, finds herself moving at $v_8 = -3\ m/s$, her *acceleration* will be calculated as +2 m/s^2. This is a POSITIVE acceleration associated with a *slow-down*.

d.) Bottom line:

i.) For +*x* motion (i.e., positive velocity):

+ avg. acc. ⇒ increase of speed
- avg. acc. ⇒ decrease of speed.

ii.) For -*x* motion (i.e., negative velocity)

+ avg. acc. ⇒ decrease of speed
- avg. acc. ⇒ increase of speed.

e.) <u>Conclusion</u>? When an object's *velocity* and *acceleration* have the same sign (i.e., are in the same direction), the body will physically speed up. When an object's *velocity* and *acceleration* have different signs, the body will slow down.

i.) In a way, this makes perfect sense. Acceleration comes only when a *net force* is applied to a body (that is, acceleration and *net force* are proportional to one another). A positive force (i.e., a *net force* directed in the *positive direction*) produces positive acceleration no matter what the velocity is. By the same token, negative force always produces negative acceleration.

ii.) In other words, if a body moving in the *-x direction* has a positive force applied to it (see Figure 4.11a to the right), we would expect the body to *slow down*. This is exactly what a POSITIVE ACCELERATION does. Likewise, you would expect the body to speed up if a negative force, hence negative acceleration, were applied (see Figure 4.11b).

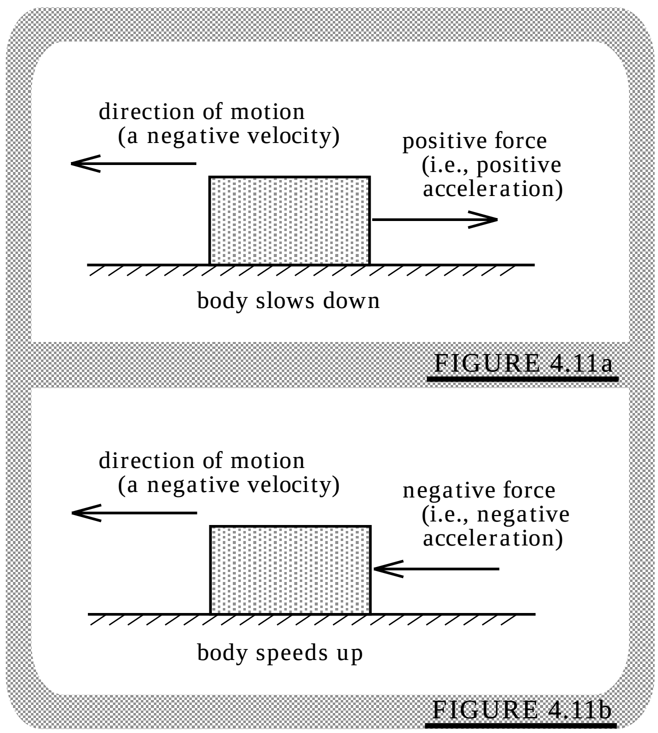

FIGURE 4.11a

FIGURE 4.11b

iii.) <u>Bottom line</u>: In both cases, our "like-directions-cause-speed-up, unlike-directions-cause-slow-down" observation is reasonable.

E.) The Kinematic Equations:

1.) To this point, we have dealt with general *position*, *velocity*, and *acceleration* functions. A special case occurs when a body is constrained to move with a CONSTANT *acceleration*.

Note: There are many constant-acceleration systems within nature. As an example: The gravitational freefall of an object near the earth's surface.

2.) With a constant acceleration, there are a number of equations that can be written that make problem-solving much easier. Collectively, these relationships are called *the kinematic equations*. They are summarized below for one dimensional motion with explanations and derivations to follow:

a.) $x_2 = x_1 + v_1\Delta t + (1/2)a(\Delta t)^2$:

i.) This states that after a time period (Δt) of constant acceleration a, an object's coordinate position x_2 equals:

ii.) Its initial position x_1 (i.e., its position at the beginning of the time interval--this initial time is usually called t_1), plus;

iii.) The *change* of position $v_1\Delta t$ due to the fact that the body has an initial velocity (i.e., v_1) at the beginning of the time interval, plus;

iv.) The additional position change $(1/2)a(\Delta t)^2$ that occurs due to the body's acceleration.

b.) $v_2 = v_1 + a\Delta t$:

i.) This states that a body's velocity v_2 after an interval Δt of constant acceleration a equals:

ii.) The body's velocity v_1 (i.e., its velocity at the beginning of the period), plus;

iii.) The increase or decrease of velocity $a\Delta t$ due to the body's acceleration.

c.) $(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$:

i.) This states that the square of a body's velocity at time t_2 (i.e., v_2) after an interval of constant acceleration a equals:

ii.) The square of the body's velocity v_1 at the beginning of the time interval (i.e., at t_1), plus;

iii.) 2 *times* the acceleration (a) *times* the change of position Δx.

d.) $x_2 = x_1 + v_{avg}\Delta t$:

i.) This states that an object's coordinate position x_2 after a period (Δt) of motion during which the average velocity has been v_{avg} equals:

ii.) The body's initial position x_1 (i.e., its position at the start of the time interval at time t_1), plus;

iii.) The additional displacement $v_{avg}\Delta t$ due to the body's motion during the time interval.

e.) $v_{avg} = (v_1 + v_2)/2$:

i.) Assuming the velocity function is linear (i.e., the acceleration is a constant), the average velocity v_{avg} between times t_1 and t_2 will simply equal the sum of the end velocities (v_1 plus v_2) divided by two.

2.) Why do we want these equations? There are times when we know, say, a body's *final velocity*, *acceleration*, and *time of acceleration*, and would like to know its *initial velocity*. We could use Calculus on the problem, but why go to all the bother when we have a kinematic equation ($v_2 = v_1 + a\Delta t$) that has all the variables we know along with the variable we are trying to determine? In other words, there are circumstances when we can short-cut the Calculus by simply using the CONSTANT ACCELERATION equations that follow from the calculus (you'll see how they follow shortly).

Note concerning the following material: The following derivations are provided so that you will have some clue as to what the variables in the kinematic equations stand for and why they relate to one another as they do. You will be expected to understand the concepts outlined below, but you will <u>not</u> be asked to reproduce the derivations. In other words, skim this material.

3.) Derivation of $v_2 = v_1 + a\Delta t$:

a.) Assume a body moves in *one-dimensional motion* under the influence of a *constant* acceleration a (as this is a one-dimensional situation, we will drop the unit vector notation). Additionally, assume that:

i.) At some initial point in time t_1, the body is positioned at x_1 and is found to be moving with velocity v_1; and

ii.) Later, at some arbitrary time t_2, the body is positioned at x_2 and is found to be moving with velocity v_2.

b.) We know the *acceleration* is the *time derivative of the velocity* function, which means we can write:

$$a = \frac{dv}{dt}$$
$$\Rightarrow dv = a(dt).$$

c.) This essentially says that a *differential* ("differential" meaning *very small*)*velocity change dv* over a *differential time interval dt* will equal the constant *rate at which the velocity changes* (i.e., the acceleration *a*) times the *time interval dt* over which the change occurs.

We can sum the velocity changes (i.e., integrate) between times t_1 and t_2. Noting that the acceleration a is a constant and, hence, can be pulled outside the integral, we can write:

$$\int_{v_1}^{v_2} dv = a\int_{t_1}^{t_2} dt$$
$$\Rightarrow v\Big|_{v_1}^{v_2} = a\left[t\right]_{t_1}^{t_2}$$
$$\Rightarrow v_2 - v_1 = a(t_2 - t_1)$$
$$\Rightarrow v_2 = v_1 + a(\Delta t) \qquad \text{(Equation A)}.$$

Note 1: It is not unusual to find this expression written in physics books as $v_2 = v_1 + at$, where the Δt has mysteriously become, simply, t. This bit of magic is justified as follows:

If the clock begins at $t_1 = 0$ and proceeds to some arbitrary time $t_2 = t$, the *change in time* is $\Delta t = (t_2 - t_1) = (t - 0) = t$. When this is incorporated into our equation, $v_2 = v_1 + a\Delta t$ becomes $v_2 = v_1 + at$.

Observation: This is *very sloppy* notation, using what looks like a particular *point in time t* in place of the time interval that belongs in the equation. Nevertheless, that is the way most physics books write it.

The moral? Be aware of what symbols mean so as not to be led astray.

Note 2: This equation is often presented as:

$$a = \frac{v_2 - v_1}{\Delta t}.$$

Note 3: This is the definition of the *average acceleration*. That makes sense. If the acceleration is constant in a system, the *average acceleration* and the *instantaneous acceleration* will be numerically equal.

4.) Derivation of $x_2 = x_1 + v_1 \Delta t + (1/2)a(\Delta t)^2$:

a.) If we assume our clock *starts* at t_1 (i.e., $t_1 = 0$), and if we set t_2 equal to an arbitrary time t so that $v_2 = v(t)$, we can rewrite *Equation A* as:

$$v(t) = v_1 + at.$$

b.) We know that the velocity $v(t)$ is the *time derivative* of the displacement function (dx/dt), which means we can write:

$$v(t) = \frac{dx}{dt}.$$

c.) Combining the two equations above we get:

$$\frac{dx}{dt} = v_1 + at.$$

d.) Briefly manipulating (i.e., not showing all the steps) yields:

$$\begin{aligned}
&\frac{dx}{dt} = v_1 + at \\
&\Rightarrow dx = (v_1 + at)dt \\
&\Rightarrow \int_{x_1}^{x_2} dx = \int_{t=0}^{t} (v_1 + at)dt \\
&\qquad = \int_{t=0}^{t} (v_1)dt + \int_{t=0}^{t} (at)dt \\
&\Rightarrow x_2 - x_1 = v_1 t + \frac{1}{2}at^2 \\
&\Rightarrow x_2 = x_1 + v_1 t + \frac{1}{2}at^2.
\end{aligned}$$

e.) Remembering that the t variable is really a Δt and $x_2 - x_1$ is Δx, this equation has an interesting graphical link. Knowing that the area under a *VELOCITY vs. TIME* graph is related to the distance traveled Δx during a given time interval, we can determine Δx using the geometry of the constant acceleration polygon shown in Figure 4.12. Doing so yields an expression that is exactly the same as the kinematics expression derived above.

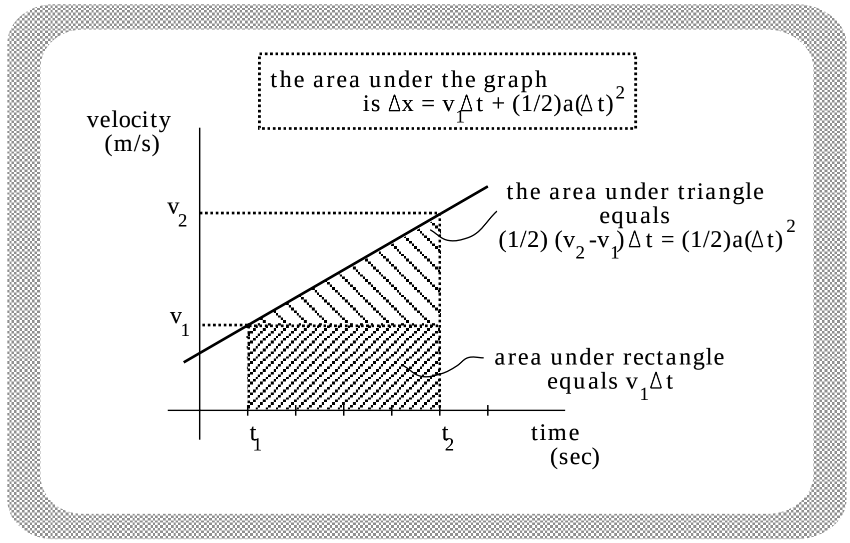

FIGURE 4.12

5.) Derivation of the expression $v_{avg} = (v_1 + v_2)/2$:

a.) Looking at the graph in Figure 4.13, it can be seen that if the acceleration is constant (i.e., the velocity is a linear function), the average velocity of the body over a time interval Δt defined by the expression $\Delta t = t_2 - t_1$ is

$$v_{avg} = \frac{v_1 + v_2}{2},$$

where v_1 and v_2 are the initial and final velocities over the interval. <u>This expression is *very rarely used*</u> but will be useful in a derivation that follows.

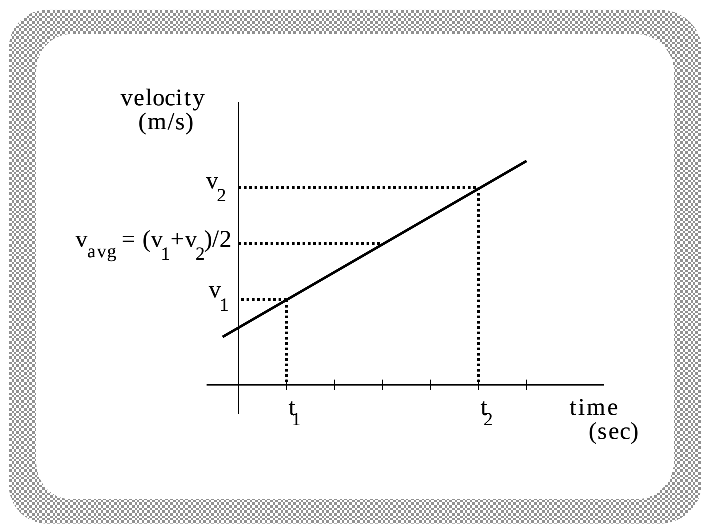

FIGURE 4.13

6.) Derivation of $\Delta x = v_{avg} \Delta t$:

a.) This is the old, "distance equals rate times time" equation you learned in the sixth grade with the *distance* term expressed as Δx and the *rate* term expressed as v_{avg}. Written in this notation, we get:

$$\Delta x = v_{avg} \Delta t.$$

Note: As is the case with all expressions having v_{avg} in them, this equation is very rarely used in the context of problem-solving.

7.) Derivation of $v_2^{\,2} = v_1^{\,2} + 2a \Delta x$:

a.) We can eliminate v_{avg} from $x_2 = x_1 + v_{avg} \Delta t$ using $v_{avg} = (v_2+v_1)/2$. Doing so yields:

$$x_2 = x_1 + [(v_2+v_1)/2]\Delta t.$$

b.) Using $v_2 - v_1 = a \Delta t$, we can solve for Δt, finding:

$$\Delta t = (v_2 - v_1)/a.$$

c.) Putting the equations from *Parts a* and *b* together, we get:

$$x_2 - x_1 = [(v_2+v_1)/2][(v_2 - v_1)/a].$$

d.) Putting $x_2 - x_1 = \Delta x$ and manipulating, we can reduce this to:

$$v_2^{\,2} = v_1^{\,2} + 2a\Delta x.$$

8.) A re-statement of the kinematic equations is presented below:

$(x_2 - x_1) = v_1 \Delta t + (1/2)a(\Delta t)^2$	(used often)
$a = (v_2 - v_1)/\Delta t$ or $v_2 = v_1 + a \Delta t$	(used often)
$(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$	(used often)
$(x_2 - x_1) = v_{avg} \Delta t$ or $v_{avg} = (x_2 - x_1)/\Delta t$	(rarely used)

$v_{avg} = (v_2 + v_1)/2$ (rarely used).

Note 1: Be careful about signs. As an example, an object moving from $x_1 = -3$ *meters* to $x_2 = -5$ *meters* does *not* have a displacement (i.e., Δx) of 2 meters. Following the math, we get $\Delta x = (x_2 - x_1) = [(-5) - (-3)] = -2$ *meters*.

Displacement is a vector; *negative* displacement means the body is moving to the left. Signs matter! Be careful with them. (The same is true whenever using velocity parameters in the equations!)

Note 2: Be sure your use of the kinematic equations is legitimate. If you are not sure whether the *acceleration* is constant, don't use them.

F.) The Kinematic Equations--Some One-Liners:

1.) A Porsche whose initial velocity is 20 m/s accelerates at 5 m/s^2 for three seconds. What is its velocity at the end of that time period?

Solution: We know the initial velocity, the constant acceleration, and the time interval over which the acceleration occurred. The equation that includes the variables we know along with the variable we need is $a = (v_2 - v_1)/\Delta t$. Using it, we get:

$$a = (v_2 - v_1) / \Delta t,$$

or

$$(5\ m/s^2) = [v_2 - (20\ m/s)] / (3\ sec)$$

$$\Rightarrow \quad v_2 = 35\ m/s.$$

2.) When our Porsche is *20 meters* to the left of a stop sign (i.e., on the *negative* side of an axis placed at the sign), it is moving with velocity of *30 m/s*. If it accelerates at a rate of $-20\ m/s^2$, how fast will it be going when at $x = -10\ m$?

Solution: We know the initial and final positions and velocities. The relationship that will do it for us is $(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$. Using it yields:

$$(v_2)^2 = (v_1)^2 + 2\ a\ [x_2 - x_1]$$

$$(v_2)^2 = (30\ m)^2 + 2(-20\ m/s^2)\ [(-10\ m) - (-20\ m)]$$

$$\Rightarrow \quad v_2 = \underline{+}22.36\ m/s.$$

Note: This kinematic equation doesn't understand *how* your Porsche is slowing down. One possibility is that you hit the brakes when at $x = -20$ *meters* (i.e., 20 meters to the left of the origin) and slide with a positive velocity (i.e., a velocity that moves to the right) through the $x = -10$ *meters* point. Another possibility is that you put the Porsche into reverse while moving in the positive direction and floor it. (This is a really dumb way to slow a car down, but it will do the trick provided you don't blow your transmission in the process.) In that case, you will slide through the $x = -10$ *meters* point on your way to a dead stop. The difference is that the car will then begin to move backwards in the negative direction passing through the $x = -10$ *meters* again but with a *negative* velocity.

The kinematic relationship you are using can't differentiate between any of these scenarios, so it deals with the problem from a purely mathematical standpoint and solves for the car's *two* potential velocities (one positive, one negative) at the $x = -10$ *meters* point.

In short, it is up to you to recognize the physical constraints of the problem and decide which sign is appropriate. Additionally, because there is no time parameter in $(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$ . . . hence no way to eliminate this ambiguity . . . this is the only kinematic relationship that does not give you the *for sure* correct sign as a part of the velocity term.

3.) In problem 2 above, how long will it take our Porsche to go from $x = -20$ *meters* to $x = -10$ *meters*?

Solution: Given the *Note* above, you'd expect *two* possible times to arise. The relationships $(x_2 - x_1) = v_1 \Delta t + (1/2)a(\Delta t)^2$ give us that. Using it yields:

$$[\;(x_2 \;-\; x_1)] = v_1 \;\Delta t + (1/2)\; a \;(\Delta t)^2$$
$$[(-10\text{ m}) - (-20\text{ m})] = (30\text{ m/s})\, t + .5\,(-20\text{ m/s}^2)\, t^2$$
$$\Rightarrow \quad t = .38 \text{ seconds and } 2.62 \text{ seconds.}$$

4.) A dragster capable of accelerating at 12 m/s^2 is given a running start at the beginning of a 400 meter race (i.e., it is allowed an initial velocity v_1). With this initial velocity, it is able to make its run in 6 seconds. What was v_1?

Solution: We know the acceleration, the distance traveled $(x_2 - x_1)$, and the time of travel. To determine the initial velocity:

$$(\;x_2 \;- x_1) = v_1 \;\Delta t \;+ (1/2)\; a \;(\Delta t)^2$$
$$(400\text{ m} - 0) = v_1(6\text{ sec}) + (1/2)(12\text{ m/s}^2)(6\text{ sec})^2$$
$$\Rightarrow \quad v_1 = 30.7\text{ m/s.}$$

5.) A dragster accelerates from rest to 110 m/s in 350 meters. What is its acceleration?

Solution: We know the initial and final velocities and the distance traveled $(x_2 - x_1)$. To get the acceleration, we could use:

$$(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$$
$$\Rightarrow \quad a = [(v_2)^2 - (v_1)^2] / [2(x_2 - x_1)]$$
$$= [(110\ m/s)^2 - (0)^2] / [2(350\ m - 0)]$$
$$= 17.29\ m/s^2.$$

G.) One More One-Dimensional Kinematics Problem--Freefall:

1.) A ball is thrown downward with an initial velocity of -2 m/s. It takes three seconds to hit the ground (see Figure 4.14). We want to determine: a.) *How high* above the ground was the ball released, and b.) *how fast* was it moving just before it hit the ground?

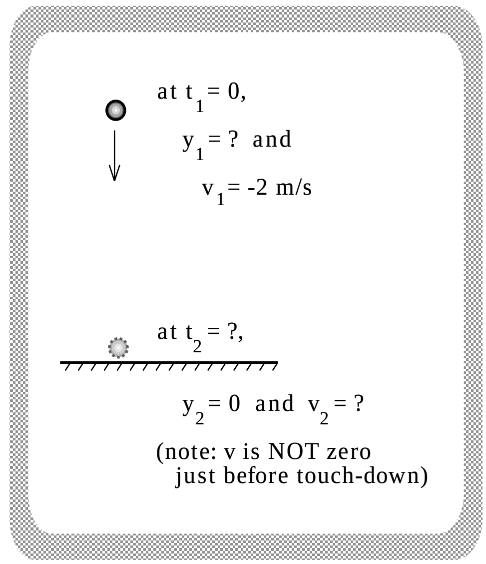

FIGURE 4.14

Solution: We know the initial velocity, the time of flight, and the acceleration (the acceleration of gravity near the earth's surface is ALWAYS approximated as $-g$, or $-9.8\ m/s^2$):

a.) To determine $y_2 - y_1$:

$$(y_2 - y_1) = v_1 \Delta t + (1/2) a (\Delta t)^2$$
$$(0 - y_1) = (-2m/s)(3\ sec) + (1/2)(-9.8\ m/s^2)(3\ sec)^2$$
$$= -50.1\ meters$$
$$\Rightarrow \quad y_1 = +50.1\ meters.$$

Note 1: Why +50.1 meters instead of -50.1 meters? Because we've placed our coordinate axis so that ground level is $y = 0$. If we had put the axis where the ball became free, our final position would have been $y = -50.1$ *meters*.

Note 2: Notice how helpful a sketch can be in visualizing a problem. Get into the habit of using sketches whenever you can.

Note 3: A temptation might have been to use $(v_2)^2 = (v_1)^2 + 2a(y_2 - y_1)$. That would be a bad move as the *velocity at ground level* is unknown (no, it is *not* zero--it is equal to the velocity *just before* touchdown).

b.) To determine v_2: With the information we now have, we could determine the velocity just before touchdown in either of two ways:

The first way:

$$\begin{aligned} (v_2)^2 &= (v_1)^2 + 2\,a\,(y_2 - y_1) \\ &= (-2 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(0 \text{ m} - 50.1 \text{ m}) \\ &= 986 \text{ m}^2/\text{s}^2. \\ \Rightarrow\ v_2 &= 31.4 \text{ m/s}. \end{aligned}$$

Note 1: As the *velocity* quantities are squared in this equation, all *negative signs* are lost in the math and the calculated value of v_2 will be a *magnitude only*. As the velocity is actually directed downward, v_2 as a vector should be written *(31.4 m/s)(-**j**)*, or *(-31.4 m/s)(**j**)*.

Note 2: It is important to notice that the particular kinematic equation used above will always yield VELOCITY MAGNITUDES ONLY.

The second way:

$$\begin{aligned} v_2 &= v_1 + a\,\Delta t \\ &= (-2 \text{ m/s}) + (-9.8 \text{ m/s}^2)(3 \text{ sec}) \\ &= -31.4 \text{ m/s}. \end{aligned}$$

Note 3: As this particular kinematic equation does not square its velocity terms, it yields both *magnitude* and appropriate *sign*. As a vector, the final solution for v_2 using this approach is *(-31.4 m/s)(**j**)*.

H.) Kinematics in Two Dimensions--Projectile Motion:

1.) Background: The net acceleration of a body moving in two dimensions can be written as $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j}$, where the acceleration components a_x and a_y may or may not be the same but are assumed to be constants. A general expression for the body's instantaneous velocity can be written $\boldsymbol{v} = v_x\boldsymbol{i} + v_y\boldsymbol{j}$, and a vector defining the body's position can be expressed as $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$.

Having formally defined these quantities, common sense tells us that a net force F_x in the *x direction* (hence an acceleration in the *x direction*) will only affect a body's motion in the *x direction*. As F_x will *not* affect the body's motion in the *y direction*, *x* and *y-type* motion must be independent of one another and must, consequently, be treated as separate entities.

2.) With the observation made above, consider a cannon positioned as shown in Figure 4.15. Its muzzle is oriented at a known angle $\theta = 30^o$ above the horizontal, and its muzzle velocity is known to be $v_1 = 100\ m/s$ (the *muzzle velocity* denotes the magnitude of the projectile's velocity as it leaves the cannon). If the cannonball becomes free at a known height $y_1 = 2\ meters$, and if it lands on a plateau whose height is $y_2 = 80\ meters$, determine:

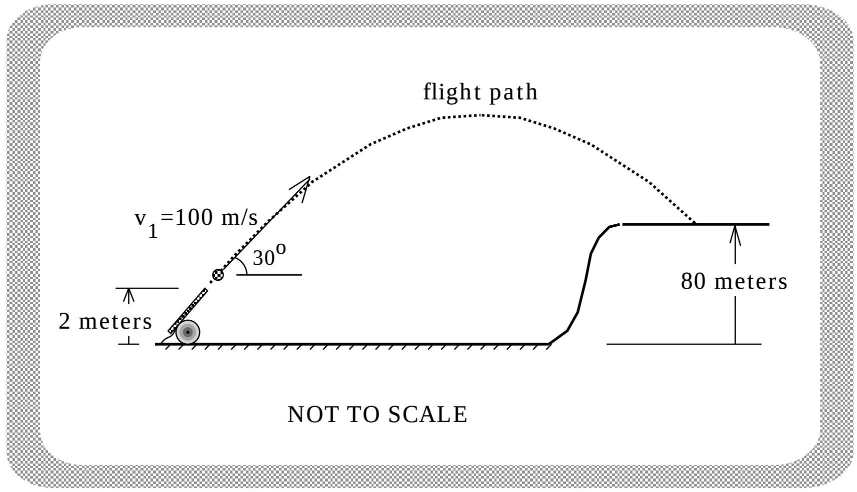

FIGURE 4.15

2a.) The time of flight Δt;

2b.) The final horizontal position x_2 of the ball at touchdown;

2c.) The velocity $\mathbf{v}_{top}$ of the cannonball at the top of its flight;

2d.) The cannonball's maximum height y_{top}; and

2e.) The velocity $\mathbf{v}_2$ of the cannonball just before its touch-down on the plateau.

3.) <u>Solutions:</u>

a.) Preliminary TIME OF FLIGHT note: Let's assume you have been sent to a point <u>down-range of the cannon</u> (see Figure 4.16a for your positioning). You have been provided with a special flight-sensing-scope that allows you to watch the cannonball's motion as it comes out of the cannon and

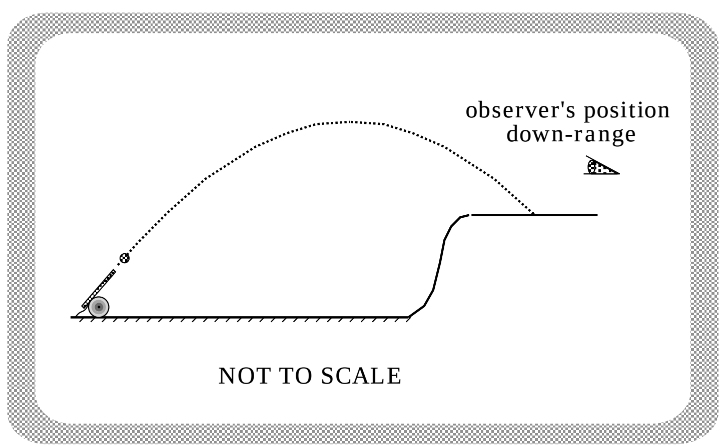

FIGURE 4.16a

proceeds on its path (you are obviously far enough away so you won't get hit by the projectile when it comes down). Additionally, let's assume that the device ruins your depth-perception (that is, you can see the ball but you don't get the feeling that it is coming toward you). From your perspective, how will the cannonball's motion look?

Reflection suggests that the cannonball will appear to rise straight upward, reach some maximum height, stop for a moment, then proceed back down toward the ground (see Figure 4.16b). Further consideration suggests the ball's initial velocity will equal the *y component* of the ball's muzzle velocity $(v_1 \sin\theta = 100 \sin 30 = 50 \text{ m/s})$.

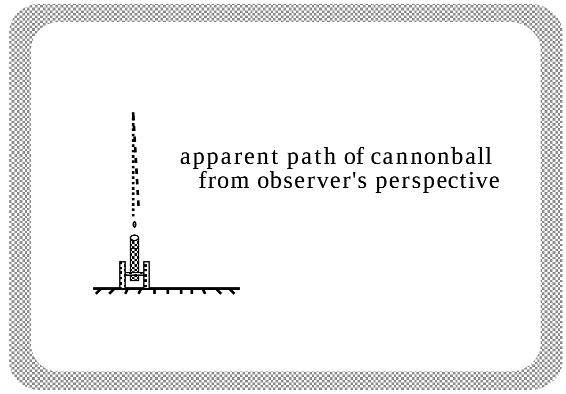

FIGURE 4.16b

From a different perspective, the cannonball's motion will exactly mimic that of a basketball thrown from *y=2 meters* directly upward with velocity 50 m/s released just as the cannonball leaves the muzzle (see Figure 4.16c).

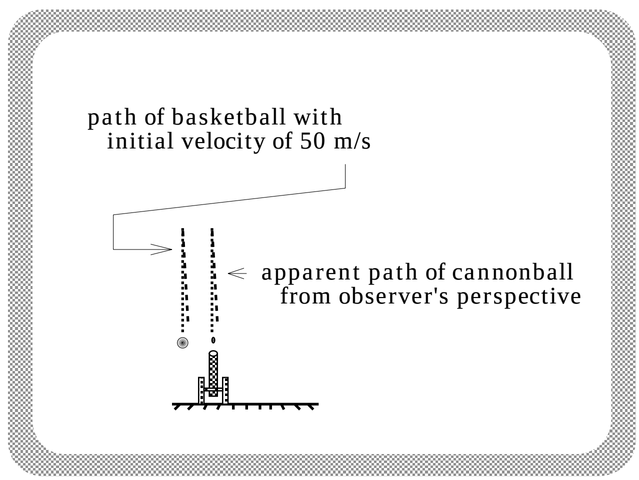

FIGURE 4.16c

We know how to use our kinematic equations to analyze the one-dimensional motion of a basketball thrown directly upward; we can use those same equations to determine the time-of-flight Δt required for either the basketball or the cannonball.

Specifically:

i.) Both balls begin at $y_1 = 2 \text{ meters}$;

ii.) Both balls rise, then fall back to $y_2 = 80 \text{ meters}$;

iii.) The initial velocity upward is $v_{1,y} = v_1 \sin\theta = (100 \text{ m})(\sin 30^o)$ $= +50 \text{ m/s}$; and

iv.) The touchdown velocity is $v_{2,y} = ?$;

v.) Acceleration in the *y direction* is due to gravity, or $a_y = -g = -(9.8\ m/s^2)$; and

vi.) The time of flight is $\Delta t = ?$

b.) To determine <u>Question 2a</u>--*Time of Flight*: The kinematic equation that will allow us to solve for the time-of-flight Δt, given the initial and final y positions, the y acceleration, and initial y velocity (see Figure 4.17), is:

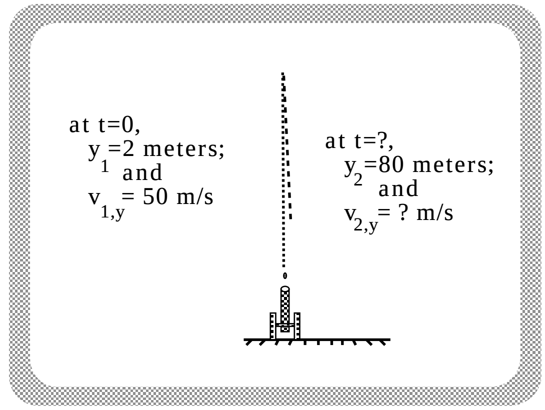

FIGURE 4.17

$$(y_2 - y_1) = \quad v_{1,y} \quad \Delta t + (1/2)\, a_y\, (\Delta t)^2$$
$$(y_2 - y_1) = (v_1 \sin\theta)\,\Delta t + (1/2)(-g)(\Delta t)^2$$

$$\Rightarrow\ (80\text{ m} - 2\text{m}) = (100\text{ m/s})(\sin 30^o)\,\Delta t\ + .5(-9.8\text{ m/s}^2)(\Delta t)^2.$$

Replacing Δt with t for simplicity, we get:

$$4.9t^2 - 50t + 78 = 0.$$

Using the quadratic formula (the solution for t in $at^2 + bt + c = 0$ is $t = [-b \pm (b^2 - 4ac)^{1/2}] / 2a$ . . .), we get

$$t = \{-(-50) \pm [\ (-50)^2 - 4\ (4.9)\ (78)\ ]^{1/2}\ \} / 2\ (4.9)$$

$$= 1.92 \text{ and } 8.28 \text{ seconds.}$$

Note: There is nothing wrong with the fact that the quadratic equations yields two solutions to this problem. The ball will be at height $y_2 = 80$ *meters* twice--once as it moves upward toward its maximum height and once on its way back down. We're interested in the time it takes to come back down to $y_2 = 80$ *meters*, so we will take the larger time of 8.28 seconds.

c.) Preliminary note to DISTANCE TRAVELED problem: Let us assume you have been placed in a helicopter and stationed high above the cannon range looking down over it. Again, you have the special flight-sensing-scope, and again you have no depth-perception when using it. From this new perspective, how will the *cannonball's* motion look (assuming you can ignore parallax problems)?

In this case, it will appear to be moving along a straight line in the *x direction*, and it will appear to be moving with a *constant velocity*. This makes sense. There are no forces acting in the horizontal which means there will be nothing to accelerate the body in the *x direction* (we are assuming there is no air-friction or wind in the system). The cannonball's *velocity* in the *x direction* will always be the *x component* of the muzzle velocity ($v_1 \cos\theta = 100 \cos 30 = 86.6$ m/s). In fact, the cannonball's motion will exactly mimic that of a car driving at a constant velocity of 86.6 m/s along the side of the range. Using our kinematic equations for the projectile's *x-type* motion, we know that

i.) $x_1 = 0$;

ii.) $x_2 = ?$

iii.) The initial velocity will be the *x component* of the muzzle velocity, or $v_{1,x} = v_1 \cos\theta = (100\ m)(\cos 30) = +86.6\ m/s$; and

iv.) $v_{2,x} = v_{1,x} = 86.6\ m/s$ (i.e., *x* velocity doesn't change)

v.) The *x direction* acceleration will be $a_x = 0$.

vi.) The time of flight (from *Part a*) will be $t = 8.28$ *seconds*.

d.) To determine Question 2b--Horizontal Displacement: The kinematic equation we will use is the same one used in the first question, but evaluated for *x-type* motion instead of *y-type* motion:

$$
\begin{aligned}
(x_2 - x_1) &= v_{1,x}\ \Delta t + (1/2)a_x(\Delta t)^2 \\
(x_2 - x_1) &= (v_1 \cos\theta)\Delta t + (1/2)a_x(\Delta t)^2 \\
\Rightarrow\ (x_2 - 0) &= (86.6\ m/s)\Delta t + .5\ (0)(\Delta t)^2 \\
\Rightarrow\ x_2 &= (86.6\ m/s)\ \Delta t \\
\Rightarrow\ x_2 &= (86.6\ m/s)(8.28\ sec) \\
&= 717\ meters.
\end{aligned}
$$

Note: How would the approach have differed if the first and second questions had been switched?

The equation $(x_2 - x_1) = v_{1,x}\Delta t + (1/2)a_x(\Delta t)^2$ would still have worked for the *x* motion, yielding $x_2 = v_1 \cos\theta\ \Delta t$, but the time of flight Δt would have been unknown. To get Δt, the equation $(y_2 - y_1) = v_{1,y}\Delta t + (1/2)a_y(\Delta t)^2$ would have had to have been evaluated for the body's *y-motion*. In other words, you would have used the same equations, but you would have written them down in the opposite order.

e.) To determine Question 2c--Velocity at Maximum Height (see Figure 4.18): In general, all velocities have components that can be written as $\boldsymbol{v} = v_x\boldsymbol{i} + v_y\boldsymbol{j}$. At the top of the flight-path, $\boldsymbol{v}_{top} = v_{x,top}\boldsymbol{i} + v_{y,top}\boldsymbol{j}$.

We know that at the cannon-ball's peak:

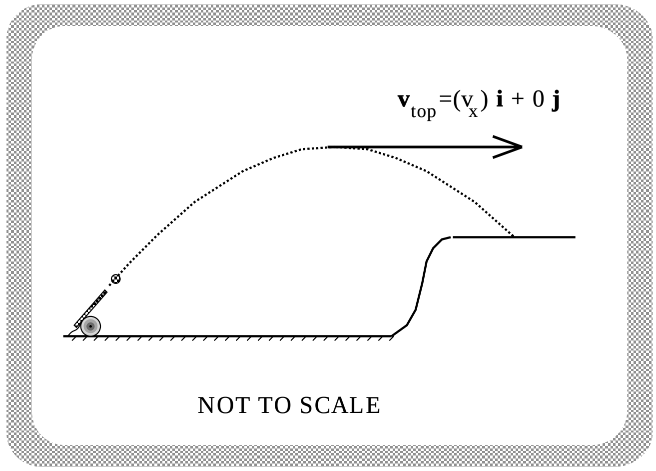

FIGURE 4.18

i.) The ball will have no vertical motion at all (that is what it means to be at the top of the path). Conclusion: $v_{y,top} = 0$.

ii.) The cannonball's horizontal (*x-type*) velocity will be as always, hence $v_{x,top} = 86.6\ m/s$.

iii.) Putting it all together, $\boldsymbol{v}_{top} = (86.6\ m/s)\boldsymbol{i} + 0\boldsymbol{j}$.

f.) Solution to Question 2d--Maximum Height: The cannonball's distance above the ground (its height) is related solely to its *y-type motion*. We have already noticed that $v_{y,top} = 0$ (i.e., the ball stops in the vertical when it reaches the top of its flight). That, coupled with the fact that we know that $a_y = -g = -9.8\ m/s^2$ and $v_{y,1} = +50\ m/s$, allows us to use $(v_{y,top})^2 = (v_{1,y})^2 + 2a_y(y_{max} - y_1)$ to solve for y_{max}. Doing so yields:

$$(v_{y,top})^2 = (v_{1,y})^2 + 2\ a_y\ (y_{max} - y_1)$$
$$(0)^2 = (50\ m/s)^2 + 2\ (-9.8\ m/s^2)\ (y_{max} - 2\ m)$$
$$\Rightarrow\ y_{max} = 129.6\ meters.$$

g.) Solution to Question 2e--Velocity Just Before Touch-down: The velocity of the cannonball just before touchdown will have a form $\boldsymbol{v} = v_{2,x}\boldsymbol{i} + v_{2,y}\boldsymbol{j}$ (see Figure 4.19). From all we've said to this point:

i.) It should be obvious that the x *component* will be the same as always--86.6 m/s.

ii.) The y *component* of the velocity will require the use of the equation $(v_{2,y})^2 = (v_{1,y})^2 + 2 a_y (y_2 - y_1)$ evaluated for y motion between $y_1 = 2$ *meters* and $y_2 = 80$ *meters*. Doing so yields:

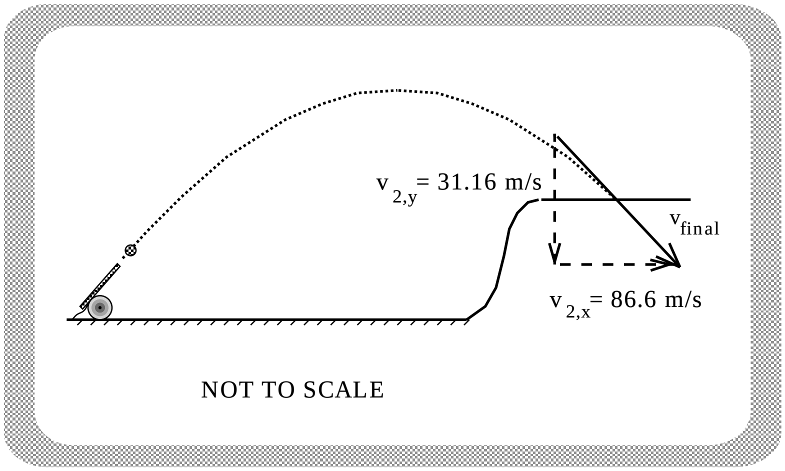

FIGURE 4.19

$$(v_{2,y})^2 = (v_{1,y})^2 + 2 a_y (y_2 - y_1)$$
$$= (50 \text{ m/s})^2 + 2 (-9.8 \text{ m/s}^2) (80 \text{ m} - 2 \text{ m})$$
$$= 971.2 \text{ m}^2/\text{s}^2$$
$$\Rightarrow v_{2,y} = 31.16 \text{ m/s}.$$

iii.) Conclusion: $\mathbf{v_2} = (86.6 \text{ m/s})\mathbf{i} + (-31.16 \text{ m/s})\mathbf{j}$.

Note: The equation used to determine $v_{2,y}$ yields velocity magnitudes only. You have to put the *negative sign* in manually after noticing that the y motion should be downward at the point of interest (see Figure 2.31).

4.) Bottom line on two-dimensional motion:

a.) Treat each direction as an entity with its own set of kinematic equations;

b.) When asked for "distance traveled in the *x direction*," think

$$(x_2 - x_1) = v_{1,x} \Delta t + (1/2)a_x(\Delta t)^2$$

with $a_x = 0$. Use this in conjunction with the same equation evaluated in the *y direction*. The time variable will allow you to link the two equations (it takes the same amount of time to go the horizontal distance as it does to go up, then down to the final vertical position).

c.) When asked to determine maximum height, think vertical motion and the equation

$$(v_{2,y})^2 = (v_{1,y})^2 + 2\ a_y\ (y_{max} - y_1)$$

with the y_{max} velocity (i.e., $v_{2,y}$) equal to zero.

d.) Be careful not to confuse *x-type* acceleration with *y-type* acceleration, especially for freefall problems (one is ZERO while the other is -*g*).

QUESTIONS

Note: Don't get hung up on Question #4.2. Understanding how to think about graphical information is important, but not as important as knowing the basic definitions and learning how to use the kinematic equations.

4.1) A sprinter runs around a 440 meter circular track in 49 seconds.

a.) What is her average speed?

b.) What is her average velocity?

c.) Can you tell anything about her instantaneous velocity 5 seconds after the start?

4.2) A student turns in the graph shown in Figure I without bothering to label the vertical axis. The graph is related to the motion of a tricycle, but all you know for sure is that at t = 1 second the trike is moving with an approximate velocity of -1 m/s. Is the graph a *position versus time* graph, a *velocity versus time* graph, or an *acceleration versus time* graph? Explain briefly.

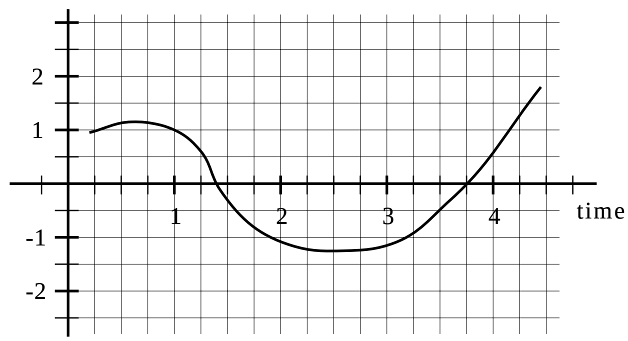

FIGURE I

4.3) Figure II is a *velocity versus time* graph for the motion of an ant moving in one dimension across the floor. Assuming you don't explicitly know the velocity function:

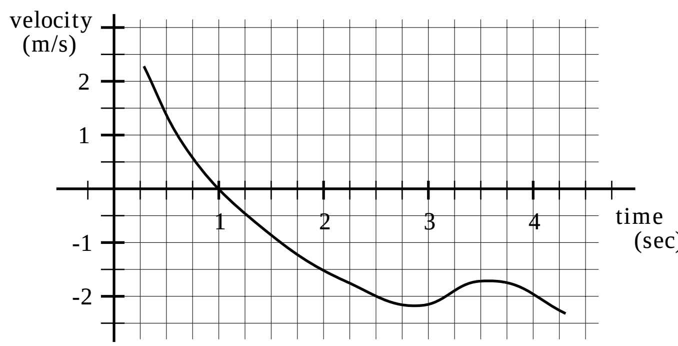

FIGURE II

a.) What is the ant's approximate displacement between times $t = .5$ *second* and $t = 3$ *seconds* (eyeball it off the graph--this is not a Calculus problem!)?

b.) What is the ant's average velocity between times $t = .5$ *seconds* and $t = 3$ *seconds*? (This is a bit off-the-wall, more of a definition/use-your-head question; if you don't see it, don't spend a lot of time on it.)

c.) What is the ant's velocity at $t = .5$ *seconds*? . . . at $t = 3$ *seconds*?

d.) What is the ant's acceleration at $t = .5$ *seconds*? . . . at $t = 3$ *seconds*?

e.) When is the ant moving in the *+x direction*?
f.) When is the ant standing still?
g.) When is the ant's acceleration approximately zero?
h.) When is the change of the ant's acceleration zero?

4.4) A body moves under the influence of a velocity function given as:

$$\mathbf{v}(t) = (3e^{-1.5kt}\mathbf{i} - 4k_1 t\mathbf{j}) \text{ m/s}.$$

Assuming both k and k_1 have magnitudes of *one* and the appropriate units:

a.) Determine the velocity of the body at $t_1 = 2$ *seconds*.

b.) Determine a general expression for the acceleration of the body as a function of time.

c.) Determine the acceleration of the body at time $t_1 = 2$ *seconds*.

d.) Could you have used kinematics to do any of the above problems? Explain your response.

e.) Determine a general expression for the position of the body as a function of time. As this will have components, call it ***r(t)***. (Hint: Do this in pieces; that is, use $v_x(t)$ to determine an expression for $x(t)$, then use $v_y(t)$ to do the same for $y(t)$).

f.) Determine the displacement of the body between times $t_1 = 2$ *seconds* and $t_2 = 3.5$ *seconds*.

g.) Assuming the body is at $x_1 = -.1$ *meters*, $y_1 = -8$ *meters* at time $t = 2$ *seconds*. *Without* using the displacement function *r(t)* derived above, what is its position coordinate at time $t = 3.5$ *seconds*?

h.) For the amusement of it, determine *the jerk* of the system.

i.) What are the units of k?

4.5) Bats at Carlsbad Caverns leave the cave at dusk in search of food. When they return at dawn, they fly over the cliff face that supports the cave entrance, fold their arms and legs, then plummet like rocks until a few meters above the floor of the cave entrance where they spread out the skin membranes between their arms and legs and pull out of the dive. Assuming they drop from a height of 100 meters and do not open their leg/wings until they are 3 meters above the floor:

a.) Ignoring air friction, how fast (i.e., the magnitude of their velocity) are they moving by the time they pull out of the freefall?

b.) If their vertical velocity essentially drops to zero as they move from 3 meters to 1 meter above the floor (i.e., during the time period in which they pull out of the freefall), what is their vertical "pull-out" *acceleration*?

c.) How long does it take them to execute their pull-out?

4.6) A particle moving in one dimension has a position function defined as:

$$x(t) = bt^4 - ct.$$

Assuming $b = 6\ m/s^4$ and $c = 2\ m/s$:

a.) At what point in time does the particle change its direction along the x axis?

b.) In what direction is the body traveling when its acceleration is 12 m/s^2?

4.7) A stunt-woman freefalls from rest. She is observed to be moving 25 m/s at a particular point in time (call her *position* at that point in time *Point A*).

a.) How far will she have fallen 2 seconds after passing *Point A*?

b.) How fast will she be moving 2 seconds after passing *Point A*?

4.8) One car moving with a constant velocity of 18 m/s passes a second car initially moving at 4 m/s. As it does, the second car begins to accelerate at a rate of 6 m/s^2.

a.) How long does it take the second car to catch the first car?

b.) How far do the cars travel during the time interval required for the second car to catch the first car?

c.) What is the second car's velocity as it passes the first car?

d.) What is the second car's average velocity during the period required for it to pass the first car?

e.) How long will it take the second car to reach 100 m/s?

4.9) A falling rock takes .14 seconds to pass from the top to the bottom of a 1.75 meter tall window in a multi-story building.

a.) What is the velocity of the rock when at the top of the window?

b.) Assuming the rock is given an initial downward velocity of 7 m/s when released at the top of the building, what is the distance between the top of the building to the *bottom* of the window?

c.) If the rock were *not* given an initial velocity of -7 m/s but instead started from rest, how would its acceleration as it passed by the top of the window have changed from the originally stated problem?

4.10) You are driving a car that can accelerate at 3 m/s^2 (it's a Nash Rambler) and can brake at 3 m/s^2. You approach an intersection that is 18 meters wide. The light turns yellow. It stays yellow for 1.2 seconds before turning red. If you accelerate, you must make it through the intersection before the light turns red to be safe. If you brake, you must stop before reaching the cross-walk-restraining-line to be safe.

Tough as it may be to believe, there is a range of distances between which you will neither be able to successfully accelerate nor brake and still be safe. Assuming you are moving 40 m/s (about 80 mph--ouch), and assuming your *reaction time* is zero (that is, you accelerate or brake just as the light turns yellow), the following will allow you to determine that range.

a.) Pedal to the metal, what is the farthest you can be from the restraining line and still be able to accelerate through the intersection before the light turns red?

b.) Braking like mad, what is the closest you can be to the restraining line and still be able to come to a grinding halt before going over the restraining line? (Note that the *slide time* does not have to be 1.2 seconds--you can still be sliding after the light turns red just as long as you don't ultimately go over the restraining line.)

c.) In conclusion, what are the *you're going to die no matter what* limits?

4.11) A 3-meter-tall elevator accelerates at a rate of 1.5 m/s^2 when it's working properly. After a shaky start, it is found to be moving with a velocity of 3.4 m/s just as its *floor* passes a point (call this *Point A*) 4 meters above the ground. As it passes this point, a bolt in the ceiling of the elevator comes loose and freefalls to the elevator's floor.

a.) Determine the bolt's maximum height above the ground during its freefall.

b.) How long did it take for the bolt to meet the floor?

c.) What was the bolt's net displacement during the freefall?

d.) What was the bolt's velocity just before striking the floor?

4.12) A batter strikes a baseball 1.3 meters above the plate. The ball leaves the bat at an angle of 50^o with a velocity of 41 m/s.

a.) How long will it take for the ball to touch down in the outfield?

b.) How far (horizontally) will the ball travel before touch down?

c.) How high will the ball travel during the flight?

d.) What will the ball's velocity be just before touch down?